

Illinois Community College Board
Adult Education & Literacy

**Illinois ABE/ASE
Mathematics
Model Curriculum
NRS Level 4**

April 2017
REVISED June 2021

Acknowledgements

The Illinois ABE/ASE Mathematics Model Curriculum was adapted from curriculum developed by the Black Hawk College Adult Education and Family Literacy program.

Thank you to the following for their contributions:

Professor Connie Kappas, Adult Education Department Chair

Instructor Gail Grigg

Adjunct Instructor Sharon Casillas

Adjunct Instructor Ann O'Leary

In June of 2021 the same curriculum was updated and revised by a committee of Math Specialists in the state of Illinois.

For the purpose of compliance with Public Law 101-166 (The Stevens Amendment), approximately 100% federal funds were used to produce this document.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)

4.RP.1 / 4.RP.2 / 4.RP.3 / 4.RP.4

Essential Understandings:

- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percent.
- Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.).
- Rates, ratios, percentages and proportional relationships express how quantities change in relationship to each other and can be represented in multiple ways.
- Rates, ratios, percentages and proportional relationships can be applied to multi-step ratio and percent problems along with other problem solving situations such as interest, tax, discount, etc.

Essential Questions:

- When is it useful to be able to relate one quantity to another?
- How can models illustrate proportional relationships?
- How can proportional relationships be used to solve ratio and percent problems?
- How can scale drawings be used to compute actual lengths and area?

Student will be able to:

(what does mastery look like)

- Solve real world and mathematical problems using ratio and rate reasoning by making tables of equivalent ratios connecting quantities with whole number measurements, finding missing values for each table of coordinate pairs, and plotting them on a coordinate plane.
- Use ratio and rate reasoning to solve unit rate problems that include unit price and a constant speed.
- Use ratio and rate reasoning to find a percent of a quantity as a rate per 100 and to find the whole given a part and the percent.
- Use ratio and rate reasoning to convert measurement units, manipulate units and transform units while multiplying or dividing quantities.
- Calculate unit rates associated with ratios of fractions, lengths, and areas in like or different units.
- Identify and show proportional relationships between quantities by determining whether two quantities are in a proportional relationship.
- Identify and show proportional relationships between quantities by identifying the proportional constant in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Identify and show proportional relationships between quantities by solving equations.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Identify and show proportional relationships between quantities by explaining in writing what a point (X,Y) on a graph of a proportional relationship means in terms of a situation, paying particular attention to the points $(0,0)$ and $(1,r)$ where r is the unit rate.
- Solve multistep ratio and percent problems by using proportional relationships.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes: on rates, ratios, percentages and proportional relationships, including: solving multi-step ratio and percent problems such as interest, tax, discount, unit price etc. (See class activities)
- Possible Texts:
 - *Ratios, Proportions and Percents* (2016). McGraw Hill/College and Career Readiness Practice Workbook.
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement* (2014). New Readers Press/Pre-High School Equivalency

Other Evidence:

- Monitor math journal entries
- Facilitate and monitor group work
- Facilitate and monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students bring in different cookie, cereal boxes, juice containers, or other food items to calculate price per unit.
- Calculate difference in gas used, (per gallon), going at a constant rate up a hill verse going at the same rate over a flat mile.
- Have students survey online shopping to calculate the final cost of sale items of their choice, including sales tax.
- All students should record their findings of math activities in their math journals.
- Use a walking field trip to estimate time in minutes based on distance and speed. Have students work in groups of four, as some may walk slower or faster.

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction interactive websites/illustrations, Study Stack – vocabulary support, etc. (See below)
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Practice work sheets-teacher made
- Ads for gasoline per gallon
- Various items from the grocery store
- Calculators
- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
- *EMPower Math, Keeping Things in Proportion: Reasoning with Ratios. Student Edition.* (2011). Contemporary/McGraw-Hill.
- *Possible Texts:*
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents, and Measurement* (2014). New Readers Press/Pre-High School Equivalency
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kahn Academy- <https://www.khanacademy.org/>
- Teaching Ideas— www.teachingideas.co.uk
- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- Cool Math— www.coolmath.com
- Math Planet— www.mathplanet.com/
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

THE NUMBER SYSTEM (NS)

4.NS.1 / 4.NS.2 / 4.NS.3 / 4.NS.4 / 4.NS.5 / 4.NS.6 / 4.NS.7 / 4.NS.8 / 4.NS.9

Essential Understandings:

- Rational numbers use the same attributes as whole numbers.
- The quotative (making groups of a certain size) and partitive (sharing equally or dealing out) types of division and measurement are applied to numbers within the real number system (fractions, decimals, integers and rational and irrational numbers).
- Quantities having more or less than zero are described using positive and negative numbers.
- Number lines are visual models used to represent the density principle: between any two whole numbers are many rational numbers, including decimals and fractions.
- The rational numbers can extend to the left or to the right on the number line, with negative numbers going to the left of zero, and positive numbers going to the right of zero.
- The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.
- Graphing objects in a four-quadrant graph can provide ways to measure distances
- Rational numbers can be represented with visuals (including distance models), language, and real-life contexts.
- There are precise terms and sequences to describe operations with rational numbers.
- Every rational number has a decimal expansion.
- Properties of operations with whole and rational numbers also apply to all real numbers.
- Absolute value is a number's distance from zero (e.g., $|-3| = 3$.)

Essential Questions:

- How are various operations (addition, subtraction, multiplication and division) represented, interpreted, and related to realistic situations and to other operations?
- How are positive and negative numbers used?
- How do rational numbers relate to integers?
- What can be modeled on the coordinate plane?
- Why are quantities represented in multiple ways?
- How can quantities be represented and what is the rationale for selecting a specific representation?
- What does the absolute value of a number represent?

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

Student will be able to:

(what does mastery look like)

- Demonstrate understanding of fractions by analyzing and solving quotients of fractions and solving word problems involving division of a fraction by a fraction.
- Show that positive and negative numbers represent quantities in real-world contexts and can be used together to describe quantities of opposite directions or values.
- Show a rational number as a point on the number line.
- Use previous knowledge of rational numbers to extend number line diagrams and coordinate axes to include negative numbers.
- Identify that the opposite of the opposite of a number is itself and identify that numbers on a number line with opposite signs are on opposite sides of 0.
- Recognize and plot the location of ordered pairs containing signed numbers in quadrants on coordinate planes, and show that the position of points of two ordered pairs differing only by sign are related by reflections across one or both axes.
- Indicate and plot the position of integers and other whole numbers on vertical or horizontal line diagrams and indicate and plot ordered pairs on a coordinate plane.
- Demonstrate understanding of ordering and absolute value of rational numbers by interpreting expressions of inequality of two numbers in terms of their relative positions on a number line diagram.
- Demonstrate understanding of ordering and absolute value of rational numbers by expressing statements of order for rational numbers in real world situations.
- Demonstrate understanding of ordering and absolute value of rational numbers by recognizing that the distance from zero of a rational number is the absolute value of the number and interpreting absolute value in real world situations as magnitude for a positive or negative quantity.
- Demonstrate understanding of ordering and absolute value of rational numbers by distinguishing between comparisons of absolute value from statements of order of rational numbers.
- Graph points in all four quadrants of the coordinate plane using prior knowledge of coordinates and absolute value to find the distance of points with the same first or second coordinate.
- Use prior knowledge of operations to add and subtract rational numbers and graph results on a line diagram.
- Apply and extend previous understanding of addition and subtraction of rational numbers by describing situations in which opposite quantities combine to make 0.
- Apply and extend previous understanding of addition and subtraction of rational numbers by showing understanding of $p + q$ as the number

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.

- Demonstrate that a number and its opposite are additive inverses, so their sum is 0, and interpret sums of rational numbers in real-world contexts.
- Apply and extend previous understanding of addition and subtraction of rational numbers by demonstrating the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Demonstrate previous knowledge of addition and subtraction of rational numbers by applying properties of operations as strategies to add and subtract rational numbers.
- Use the properties of multiplication, division and fractions to multiply and divide rational numbers.
- Demonstrate the distributive property with signed numbers and apply to real world situations.
- Apply division to integers with divisors other than zero, recognizing the quotient of integers is a rational number, and apply to real life situations.
- Using prior knowledge of properties of operations, multiply and divide rational numbers.
- Use long division to convert a rational number to a decimal, recognizing the decimal form of rational numbers ends in 0 or repeats.
- Use the four operations with rational numbers to solve real-world and mathematical problems, including the ability to manipulate complex fractions.
- Demonstrate understanding that every number has a decimal expansion of some kind and that rational numbers end in 0's or eventually repeat.
- Compare the size and value of irrational numbers by using rational number approximations, locate them on a number line diagram, and provide an approximate estimation of value.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes (i.e., given figures for a family budget, determine which of a list of bills can be paid without creating a deficit and identify deficit amount if all were paid)
- Text: Steck-Vaughn and Contemporary's Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter/unit reviews/tests from core texts such as New Readers Press *Math Sense*. (See below)

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students work in groups of four to reduce a recipe and record new measurements
- Write a real-life math problem involving hourly wages with weekly pay
- Have students use a four-function calculator to check answers
- Use integers to keep track of scores in an interactive football game
- Analyze temperatures over the last four years. Students work in groups choosing different areas from any continent
- Have students rewrite recipes to increase number of servings
- Use interactive technology to practice additive inverse operations
- Have each student make up Scientific Cards in two different colors, one color for standard decimal notation and the other for scientific notation
- Use cards for memory game or matching game
- Have students make a number line using irrational numbers putting them in sequential order

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology: Study Stack, websites as assigned
- One-to-one intervention
- Peer tutoring
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials

- Large number line
- Chart of unit rates, such as ounces, pounds, gallons, hours, minutes, etc.
- Geoboards
- Colored index cards
- Calculators

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

- Math fraction manipulatives
- Scientific Notation math chart
- *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
- *Possible Texts*
 - *Math 1: Whole Numbers, Decimals, Fractions, Percents and Measurement.* (2014) New Readers Press/Pre-High School Equivalency
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- Cool Math— www.coolmath.com
- Khan Academy— <http://khanacademy.org/>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- iPathways

EXPRESSIONS AND EQUATIONS (EE)

4.EE.1 / 4.EE.2 / 4.EE.3 / 4.EE.4 / 4.EE.5 / 4.EE.6 / 4.EE.7 / 4.EE.8 / 4.EE.9 /
4.EE.10 / 4.EE.11 / 4.EE.12

Essential Understandings:

- Variables within algebraic expressions are a modeling tool to use when solving real-world problems. This process demonstrates a method of describing quantitative relationships – for instance, traveling some distance (d) at a given rate of travel will take a given amount of time (t) with a constant rate.
- Solving equations is a reasoning process and follows established procedures based on properties.
- The value of any real number can be represented in relation to other real numbers such as with decimals converted to fractions, scientific notation and numbers written with exponents (e.g., $8 = 2^3$.)
- Understand the difference between an expression and an equation: expressions are simplified and equations are solved for the variable's value.
- Properties of operations can be used to add, subtract, factor, and expand linear expressions.
- Expressions can be manipulated to suit a particular purpose to solve problems efficiently.
- Mathematical expressions, equations, inequalities and graphs are used to represent and solve real-world and mathematical problems.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Properties, order of operations, and inverse operations are used to simplify expressions and solve equations efficiently.
- Unit rates can be explained in graphical representations and algebraic equations.
- The solution to a system of two linear equations in two variables is an ordered pair that satisfies both equations.
- Some equations/inequalities and systems of equations/inequalities have no solutions (parallel lines) and others have infinite solutions (same line).
- Square roots and cube roots of small perfect squares and cubes can be evaluated and/or represent solutions to the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number.
- The properties of integer exponents can generate equivalent numerical expressions.

Essential Questions:

- What is equivalence? (in reference to expressions and equations)
- How are properties of operations used to prove equivalence?
- When and how are expressions, equations, inequalities and graphs applied to real world situations?
- How can the order of operations be applied to evaluating expressions, and solving from one-step to multi-step equations?
- What are some possible real-life situations to which there may be more than one solution?
- How does the ongoing use of fractions and decimals apply to real-life situations?
- What makes a solution strategy both efficient and effective?
- How is it determined if multiple solutions to an equation are valid?
- How does the context of the problem affect the reasonableness of a solution?
- Why can two equations be added together to get another true equation?
- How can the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number be evaluated?
- What is the significance of scientific notation for very large or very small numbers within problem solving situations?

Student will be able to:

- Add, subtract, factor, and expand linear expressions with rational coefficients by applying properties of operations as strategies.
- Write, read, and evaluate expressions involving variables by showing that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. (4EE.11)

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Demonstrate understanding of expressions by rewriting an expression in a new form to clarify the problem and the way in which quantities are related to each other.
- Use tools strategically to solve multi-step real-life and mathematical problems with positive and negative rational numbers in any form, apply properties of operations as strategies to calculate, converting between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation.
- Using variables to represent real-world or mathematical quantities, construct simple equations and inequalities to solve problems by reasoning about quantities.
- Construct simple equations and inequalities to solve problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are rational numbers, and then compare an algebraic solution to an arithmetic solution and identify the sequence of operations used.
- Construct simple equations and inequalities to solve problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are rational numbers, and graph and interpret the context of the problem.
- Generate equivalent numerical expressions by understanding and applying properties of integer exponents.
- For equations such as $x^2 = p$ and $x^3 = p$, represent solutions by using square root and cube root symbols, evaluating square roots of small perfect squares and cube roots of small perfect cubes.
- Estimate very large or very small quantities and express how many times as much one is than another by using numbers expressed in the form of single digit times an integer power of 10.
- Solve problems with numbers using scientific notation and choose units of appropriate size for measurements of quantities both very large and very small.
- Demonstrate understanding of proportional relationships and lines by graphing proportional relationships, classifying the slope of a line in unit rate, and comparing two proportional relationships represented in different ways.
- Demonstrate understanding of proportional relationships and linear equations by using similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line in the coordinate plane.
- Derive $y = mx$ for a line passing through the origin, and derive $y = mx + b$ for a line intercepting the y -axis at b .
- Demonstrate understanding of single variable equations by providing examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions, and show which possibility is true by converting the given equation to a simpler form.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Analyze and solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions with the distributive property and combining like terms.
- Evaluate and solve pairs of simultaneous linear equations by understanding that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- Evaluate and solve pairs of simultaneous linear equations by solving systems of two linear equations in two variables algebraically, and estimate by graphing the equations.
- Evaluate and solve pairs of simultaneous linear equations by solving real-world and mathematical problems leading to two linear equations in two variables.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes such as use substitution to determine monthly car payments based on a four-year plan (See class activities)
- Text: Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Estimate weekly wages based on commission
- Use substitution to determine monthly car payments based on a three-year plan
- Study price of gas, calculating percentage of increase/decrease
- Write a multi-step equation based on a present hourly wage with a 10% raise. Calculate the new hourly pay
- Write an inequality based on how many hours a person need to work per week if they earn x per hour with y axis

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Using information from a real-world problem, graph prop
- Proportional relationships, interpreting the unit rate as the slope of a graph
- Estimate the population of endangered species, writing it in scientific notation
- Study price of gas, calculating percentage of increase/decrease
- Write a multi-step equation based on a present hourly wage with a 10% raise. Calculate the new hourly pay
- Write an inequality based on how many hours a person needs to make a week to satisfy a given budget
- Using information from a real-world problem, graph proportional relationships, interpreting the unit rate as the slope of a graph
- Estimate the population of endangered species, writing it in scientific notation

Learning Activities:

(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology: Study Stack, websites as assigned
- One-to-one intervention
- Peer tutoring
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Think out loud (demonstrate how to think about a problem)
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Spread sheets
- Calculators
- Multiplication boxes
- Counters
- Fictional budgets and financial records
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Math Sense 2: Algebraic Thinking, Data Analysis, and Probability.* (2014). New Readers Press.
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Kuta Software— <https://www.kutasoftware.com>
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

FUNCTIONS (F)

4.F.1 / 4.F.2 / 4.F.3 / 4.F.4 / 4.F.5

Essential Understandings:

- A function is a specific topic of relationship in which each input has a unique output that can be represented in a table.
- A function can be represented graphically using ordered pairs that consist of the input and the output of the function in the form (input, output).
- A function can be represented with an algebraic rule.
- The equation $y = mx + b$ is a straight line and that slope and y-intercept are critical to solving real problems involving linear relationships.
- Changes in varying quantities are often related by patterns that can be used to predict outcomes and solve problems.
- Linear functions may be used to represent and generalize real situations.

Essential Questions:

- How do ordered pairs on coordinate graphs help define relationships?
- What defines a function and how can it be represented?
- What makes a function linear?
- How can linear relationships be modeled and used in real-life situations?
- Why is one variable dependent upon the other(s) in relationships?

Student will be able to:

- Define a function by explaining that the graph of a function is the set of ordered pairs consisting of an input and a corresponding output.
- Compare functions by considering properties of two functions that are represented algebraically, graphically, numerically, or verbally.
- Evaluate functions by demonstrating that $y = mx + b$ represents a linear equation as opposed to functions that are non-linear.
- Model relationships between two quantities by creating a function, determine the rate of change and value of the function from a description of given (x,y) values, and interpret and find the rate of change and initial value of a function by reading from a graph.
- Model relationships between two quantities by analyzing a graph where the function is increasing or decreasing, linear or nonlinear.
- Model relationships between two quantities by sketching a graph showing the qualitative features of a function based on a verbal description.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes: interpret two growth charts
- Textbook Unit Reviews and/or exercises as quizzes and chapter tests
- Chapter/unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Compare graphs on social media usage in the last ten years
- Read growth charts, comparing American child growth to that in third world countries
- Interpret nutritional graphs indicating data from fast food restaurants

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Graph paper
- Rulers
- Graphing Calculators- Desmos and TI30Xs
- Posters containing real world graphed information, (i.e., Nutrition, financial)
- Newspaper graphs

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

- *Possible Text:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Seeking Patterns, Building Rules: Algebraic Thinking, Student Edition.* (2011). McGraw-Hill.
 - *Math Sense 2: Algebraic Thinking, Data Analysis and Probability* (2014). New Readers press
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Khan Academy— <http://khanacademy.org/>
- Kuta Software— <https://www.kutasoftware.com>
- Vocabulary support: www.studystack.com/
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

GEOMETRY (G)

4.G.1 / 4.G.2 / 4.G.3 / 4.G.4 / 4.G.5 / 4.G.6 / 4.G.7 / 4.G.8 / 4.G.9 / 4.G.10 /
4.G.11 / 4.G.12 / 4.G.13 / 4.G.14

Essential Understandings:

- Scale drawings can be applied to problem solving situations involving geometric figures.
- Geometrical figures can be used to reproduce a drawing at a different scale
- Algebraic reasoning is applied when solving geometric problems (i.e., circumference and area of a circle).
- Unit rates can be explained in graphical representation, algebraic equations, and in geometry through similar triangles.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real-life experiences.
- Properties of two-dimensional shapes are used in solving problems involving three-dimensional shapes.
- Planes that cut poly-hedra create related two-dimensional figures. Reflections, translations, and rotations are actions that produce congruent geometric objects.
- Dilations, translations, rotations and reflections can be shown using two-dimensional figures on a coordinate plane.
- A dilation is a transformation that changes the size of a figure but not the shape.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Two similar figures are related by a scale factor, which is the ratio of the lengths of corresponding sides.
- If the scale factor of a dilation is greater than 1, the image resulting from the dilation is an enlargement, and if the scale factor is less than 1, the image is a reduction; both transformations result in similar figures.
- A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of transformations.
- Two shapes are similar if the length of all the corresponding sides are proportional and all the corresponding angles are congruent.
- Congruent figures have the same size and shape (a rigid, fixed relationship). If the scale factor of dilation is equal to 1, the image resulting from the transformation is a congruent figure.
- Right triangles have a special relationship among the side lengths that can be represented by a model and a formula.
- The Pythagorean Theorem and its converse can be proven.

Essential Questions:

- Why is it important to use precise language and mathematical tools in the study of two-dimensional and three-dimensional figures?
- How do graphs illustrate proportional relationships?
- How are geometric figures used to reproduce a drawing at a different scale?
- What models on the coordinate plane are helpful for understanding and quantifying the volume of rectangular prisms?
- How does investigating figures help us build our understanding of mathematics?
- How can proportional relationships of congruent and similar figures be used to solve ratio problems?
- How are scale drawings used to compute actual lengths and area?
- What are transformations and what effect do they have on an object?
- What does the scale factor of a dilation convey?
- How can transformations be used to determine congruency or similarity?
- What angle relationships are formed by a transversal intersecting with two parallel lines?
- Why does the Pythagorean Theorem apply only to right triangles?
- How does the knowledge of how to use right triangles and the Pythagorean Theorem enable the design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads?
- How do indirect measurement strategies (using the Pythagorean Theorem) allow for the measurement of items in the real world such as playground structures, flagpoles, and buildings?
- How is the volume and/or surface area of various three-dimensional geometric objects determined?

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

Student will be able to:

- Draw, construct and describe geometrical figures by solving problems with scale drawings and computing actual lengths and areas from a scale drawing as well as reproducing a scale drawing at a different scale.
- Draw, construct and describe geometric shapes with given conditions by focusing on triangle construction from three angle or side measures and recognizing what conditions determine particular triangles, more than one triangle, or no triangle.
- Construct and describe geometrical figures by slicing three-dimensional figures, such as plane sections of right rectangular prisms or right rectangular pyramids.
- Solve real-world and mathematical problems involving measurements of angles, area, surface area, and volume through application of formulas for area and circumference of a circle as well as identifying the relationship between a circle's area and its circumference.
- Solve real-life and mathematical problems pertaining to angle measure by applying rules of supplementary, complementary, vertical and adjacent to multi-step problems with equations for unknown angle in a figure.
- Solve real-life and mathematical problems pertaining to area, surface area, and volume for two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations through identifying that lines are taken to lines and line segments to line segments of the same length.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations through observing that angles are taken to angles of the same measure.
- Demonstrate knowledge of congruence and similarity by verifying experimentally the properties of rotations, reflections, and translations by observing that parallel lines are taken to parallel lines.
- Demonstrate knowledge of congruence by showing that a pair of two-dimensional figures are congruent if the second can be obtained from the first, and describe a sequence that exhibits the congruence between the two figures.
- Use coordinates to demonstrate the effects of dilations, translations, rotations, and reflections on plane figures.
- Demonstrate knowledge of similarity by showing that a pair of two-dimensional figures are similar if the second can be obtained from the first, and describe a sequence that exhibits the similarity between the two figures.
- Develop facts about angle sums and exterior angles of triangles, angles created by parallel lines cut by transversals, and the angle-angle criterion.
- Identify and explain a proof of the Pythagorean Theorem and its converse.

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Solve real-world and mathematical problems for unknown side lengths in two- and three-dimensional geometric shapes containing right triangles by applying the Pythagorean Theorem.
- Find the distance between two points in a coordinate system by applying the Pythagorean Theorem.
- Demonstrate knowledge of formulas for volume of cylinders, cones, and spheres by applying them to and finding solutions for real-world and mathematical problems.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher-generated assignments and quizzes: Sketch geometric shapes as directed and divide in half
- Text book Unit reviews or tests and/or exercises as quizzes and chapter tests
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work
- Exit slips, math minutes, etc.

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Estimate the amount of tile needed to cover a 12ft² floor.
- Have students plan a garden in different shape and size planters, finding the volume of soil needed to fill each pot. Chart findings to compare results
- Sketch geometric shapes, extend activity by slicing them in half or thirds
- Have students create three-dimensional solid rectangular objects using Paper Mache. Then have students use their rectangular solids to calculate its total surface area *or from templates available at <https://www.math-salamanders.com/3d-geometric-shapes.html>*
- Have students make picture frames of various shapes, cutting them from stock paper. This will involve precise angle measurement and cutting to fit edges together

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Using hand sketched rotations, reflections, dilations and translations, write the effects on two-dimensional figures using coordinates
- Tie a rope to a basketball net and stretch it out to form an angle. Have students apply the Pythagorean Theorem to find the distance from the net to the ground
- Challenge students to bring in examples of cylinders, cone, and spheres. Discuss the formulas for each volume and apply them to each item brought in by students

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Advertisements for prices on tile, carpet, (based on sq. ft.), and bags of soil
- Interactive technology math sites
- Stock paper
- Protractors
- Rulers
- Scissors
- Rope for creating angles
- Calculators
- Solid three-dimensional objects that can be used to find surface area
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *EMPower Math, Over, Around, and Within: Geometry and Measurement, Student Edition.* (2011). Contemporary/McGraw-Hill.
 - *Geometry and Measurement* (2016). McGraw-Hill/College and Career Readiness Practice Workbook
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

List of Technology Resources:

- GeoGebra <https://www.geogebra.org/?lang=en>
- Khan Academy— <http://khanacademy.org/>
- Study Stack --- www.studystack.com
- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org

STATISTICS AND PROBABILITY (SP)

4.SP.1 / 4.SP.2 / 4.SP.3 / 4.SP.4 / 4.SP.5 / 4.SP.6 / 4.SP.7 / 4.SP.8 / 4.SP.9

Essential Understandings:

- Bivariate categorical data display frequencies and relative frequencies can be seen in two-way tables.
- Random sampling tends to produce representative samples and support valid inferences.
- The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- The probability of a chance event is approximated by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the approximate relative frequency given the probability.
- A probability model, which may or may not be uniform, is used to find probabilities of events.
- Various tools are used to find probabilities of compound events (including organized lists, tables, tree diagrams, and simulations).
- Written descriptions, tables, graphs, and equations are useful in representing and investigating relationships between varying quantities.
- Slope and y-intercept are keys to solving real problems involving linear relationship models of data.
- Some data may be misleading based on representation.

Essential Questions:

- Using measures of central tendency, how are data sets interpreted and analyzed?
- How are probability and the likelihood of an occurrence related and represented?
- How is probability approximated?
- How is a probability model used?

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- How are probabilities of compound events determined?
- What relationships can be seen in bivariate categorical data?
- What conclusions can be drawn from data displayed on a graph?
- What do the slope and y-intercept of a line of best fit signify on a graph? What do outliers signify?
- How can graphs, tables, or equations be used to describe patterns and predict subsequent data or outcomes?

Student will be able to:

- Summarize numerical data sets in relation to their context by reporting the number of observations.
- Summarize numerical data sets in relation to their context by describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability, as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered.
- Summarize numerical data sets in relation to their context by relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data was gathered.
- Use random sampling to draw inferences about a population with an unknown characteristic, and generate multiple samples of the same size to gauge the variation in estimates or predictions.
- Use random sampling to draw inferences about a population by using measures of center and measures of variability to draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models by demonstrating that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.
- Investigate chance processes and develop, use, and evaluate probability models by approximating the probability of a chance event by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the relative frequency given the probability.
- Investigate chance processes and develop, use, and evaluate probability models by comparing probabilities from a model to observed frequencies and if the agreement is not good, explain possible sources of the discrepancy.
- Develop a probability model by assigning equal probability to all outcomes and using the model to determine probabilities of events, as well as by observing frequencies in data generated from a chance process.
- Use organized lists, tables, tree diagrams, and simulation to find probabilities of compound events, demonstrating why the probability of a

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

compound event is the fraction of outcomes in the sample space for which the compound event occurs.

- Use organized lists, tables, and tree diagrams to find probabilities of compound events, representing sample spaces for compound events, and design a simulation generating frequencies for compound events.
- Investigate patterns of association in bivariate data by constructing and interpreting scatter plots for bivariate measurement data in order to investigate patterns of association between two quantities, classifying patterns such as clustering, outliers, positive or negative association, linear or nonlinear association.
- Investigate patterns of association in bivariate data by demonstrating why straight lines are widely used to model relationships between two quantitative variables.
- Investigate patterns of association in bivariate data by using equations of linear models to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Investigate patterns of association in bivariate data by constructing and interpreting a two-way table summarizing data on two categorical variables collected from the same subjects, using relative frequencies for rows or columns to describe possible association between the two variables.

Evidence for Assessing Learning

Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Teacher made assignments and quizzes (See class activities)
- Chapter /unit reviews/tests from core texts such as New Readers Press *Math Sense*

Other Evidence:

- Monitor math journal entries
- Monitor group work
- Monitor independent work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Have students work in groups to collect data on a topic of each group's choice. Each group should display numerical data in a different way, for example: one group could plot on a number line, another group on dot plots, and still another using box and whisker

NRS Level 4 Math

Middle Intermediate Basic Education (Grade Levels 6-7)

- Adding to this classroom activity, students must explain in their math journals the distribution on the data collected
- Collect data from students to determine associations between test scores and sleep, coffee and sleep, stress and sleep, etc.
- Plot and analyze related student attributes (i.e., bed times and wake up times, workdays per week and times eating out per week, number of children, number of hours of sleep per night)
- Survey two different populations from opposite climates and take random samples to draw informal comparative inferences. Each student can measure the center and variability for the numerical data collected
- Play a probability game using dice or playing cards. Students can roll the dice several times and keep track of the chances it produces and its frequency over time
- Students can do the same with tossing a coin. Students are to record all findings in their math journal
- A third activity will be using coins and dice for compound events.
- All activities should be recorded on tables, tree diagrams, and on organized lists
- As a whole group, plot and analyze each student's daily routines and use this information to understand and explain patterns of association in bivariate data
- Then use a linear equation to solve problems in the context of a bivariate measurement and interpret the slope and intercept
- Students then record frequencies and relative frequencies in a two-way table

Learning Activities:

(interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Interactive technology as assigned by instructor to support instruction such as: interactive websites/illustrations, Study Stack – vocabulary support
- One on one intervention
- Think out loud (demonstrate how to think about a problem)
- Peer teaching through group work
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards and games)

List of Instructional Materials:

- Newspaper (local and world)
- Chart paper, graph paper

NRS Level 4 Math Middle Intermediate Basic Education (Grade Levels 6-7)

- Interactive technology
- Dice (6-sided, 10-sided, etc.), coins, cards, rulers
- Markers and colored pencils
- *Possible Texts:*
 - *Mathematical Reasoning: Test Preparation for the 2014 GED® Test. Student Book and Workbook.* (2013). Steck-Vaughn.
 - *Math Sense 2: Algebraic Thinking, Data Analysis, and Probability* (2014). New Readers Press
 - *Common Core Basics: Mathematics.* (2013). Contemporary/McGraw-Hill.

List of Technology Resources:

- Kuta Software— <https://www.kutasoftware.com>
- Purple Math— www.purplemath.com
- CK-12- <https://flexbooks.ck12.org/pages/adult-education/#adult-basic-education>
- i-Pathways— www.i-pathways.org