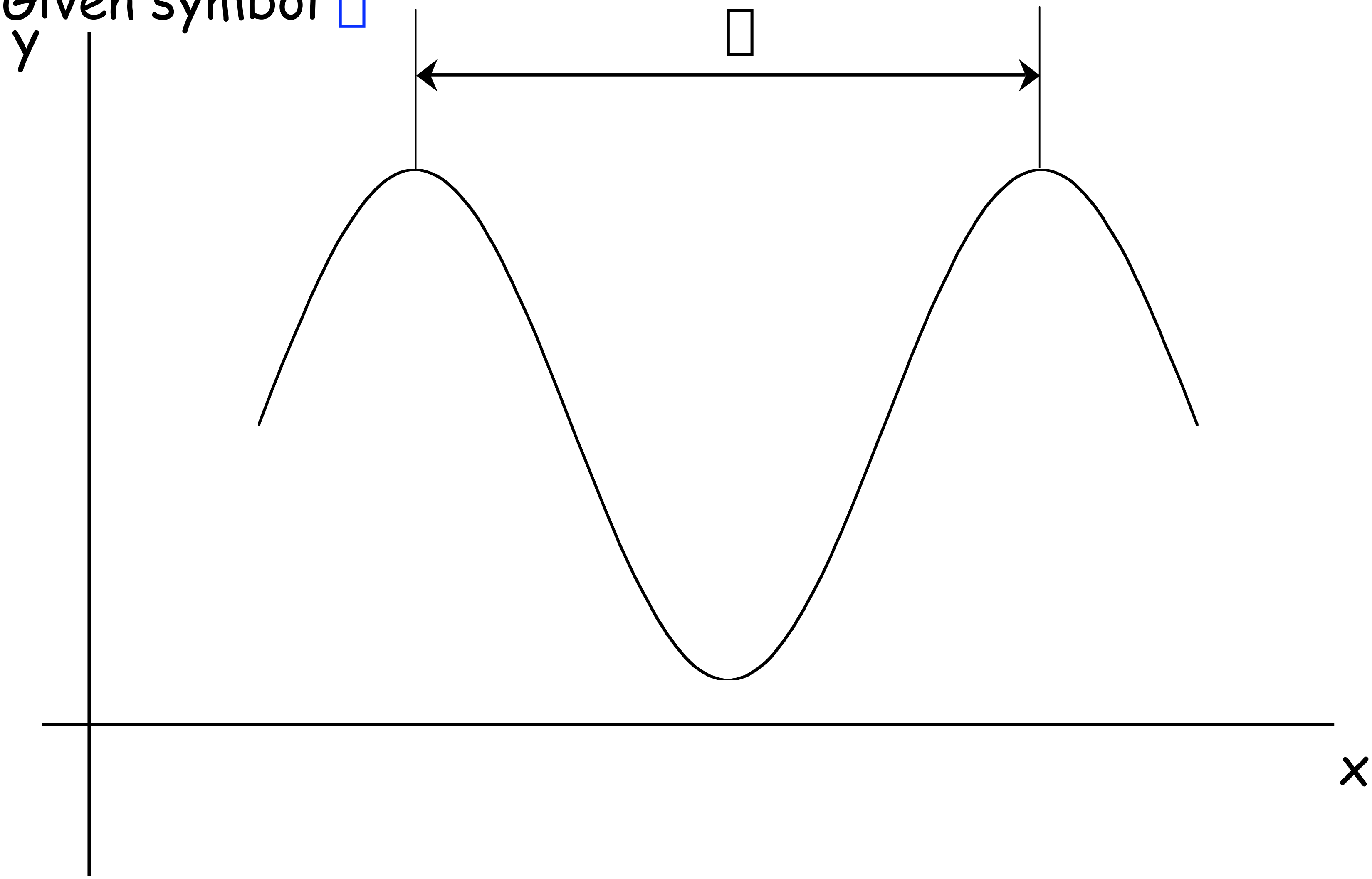
1.4 Periodic Waves

- Often have situations where wave repeats at regular intervals
  - Electromagnetic wave in optical fibre
  - Sound from a guitar string.
- These regularly repeating waves are known as periodic waves.
- Can characterize periodic waves either by the length scale, wavelength, or the time scale, period, at which they repeat.

Periodic wave in spatial domain - length scale is wavelength

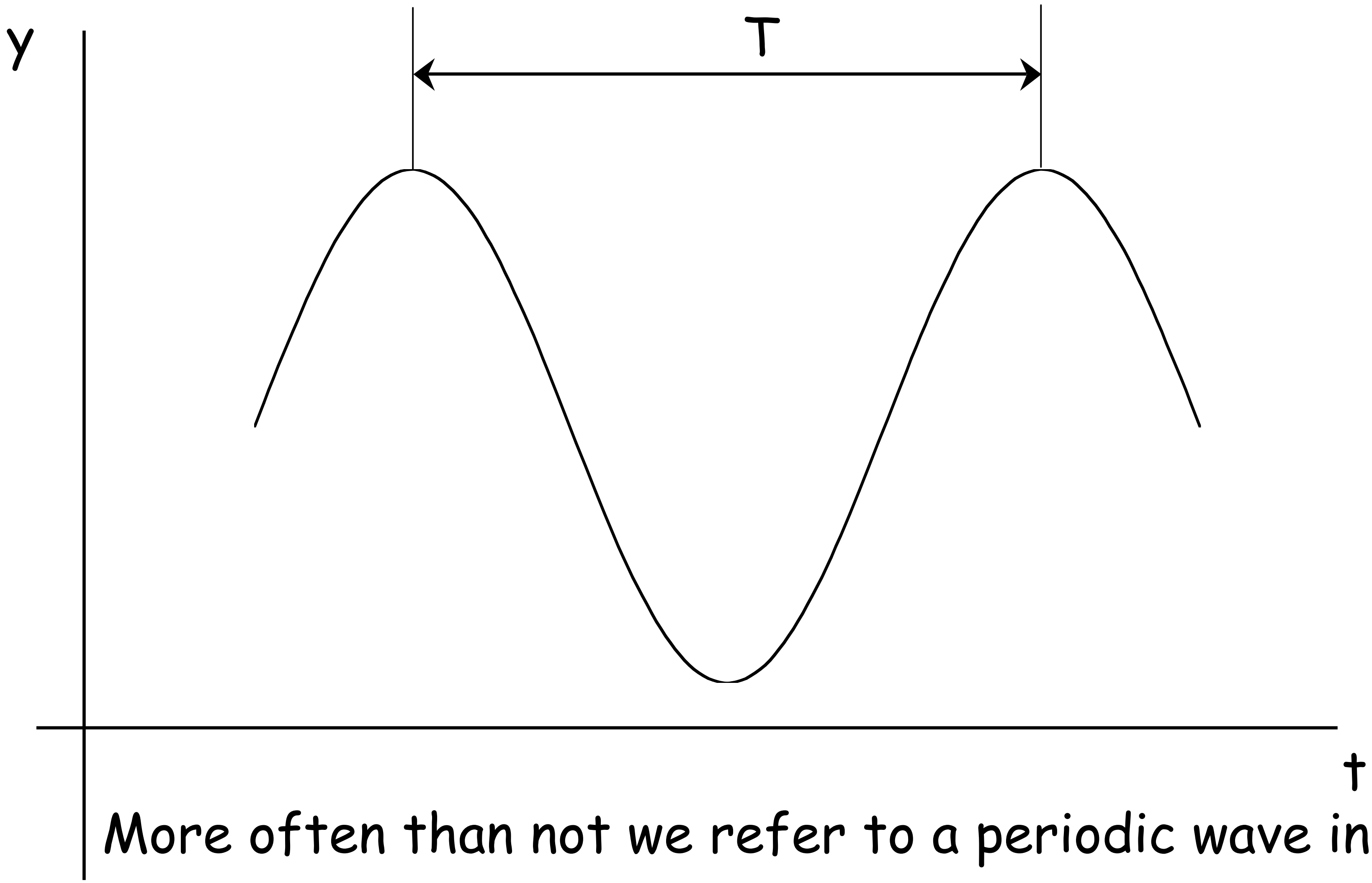
Given symbol λ



For red light (HeNe laser) λ = 632.8 nm.

For middle C on a piano λ = 1.3 m.

Periodic wave in time domain - time scale is period



More often than not we refer to a periodic wave in terms of the number of times the wave repeats in 1 second. This is the frequency, f. $f = 1/\text{period}$ so $f = 1/T$.

Wave speed for period waves

- Find that the wavelength, period and wave speed are related by the following

$$v = \frac{\lambda}{T}$$

- This can be written as

$$v = f\lambda$$

- For green light, λ = 500 nm, v = 3×10^{8} m s^{-1}, f = 6×10^{14} Hz.
- For middle C, λ = 1.3, v = 340 m s^{-1}, f = 262 Hz.

1.5 Sine Waves and Periodic Waves

- We can write the wave function for an arbitrary disturbance as

$$y(x,t) = f(x - vt)$$

  with f() describing an arbitrary function.
- For periodic waves we can use sin/cos to give functionality to the wave.
- Why sin/cos?
  - They are periodic in 2π.
  - They represent a pure colour or pure tone.
  - Complex waves can be made up from the addition of sin/cos waves - Fourier theory.

1.5 Sine Waves and Periodic Waves

- For a periodic wave the wave function is

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

- The term $2\pi/\lambda$ scales the wave to the natural period of the sin function.
- The term A gives the amplitude of the wave which is the maximum displacement of the wave.

1.5 Sine Waves and Periodic Waves

- A more elegant way of writing the wave function is

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

- The phase of the wave is kx - wt + ϕ.
- **THE PHASE IS ALWAYS MEASURED IN RADIANS.**
- The term $k = 2\pi/\lambda$ is called the wave number.
- The term $\omega = 2\pi v/\lambda \equiv 2\pi f$ and is the angular frequency.
- The term ϕ is the initial phase of the wave.

1.5 Sine Waves and Periodic Waves

- This representation of the wave function

$$y(x,t) = A\sin(kx \; \square \, \square t + \square)$$

- Is called the □ - k notation.
- The wave speed is given by

$$v = \frac{\square}{k} \equiv \frac{2\square f}{\frac{2\square}{\square}} \equiv \square f$$

1.5 Sine Waves and Periodic Waves

- The wave function for a wave is given by

$$y(x,t) = 0.02\,sin(0.4x \square 50t + 0.8)m$$

- Find (a) the amplitude, (b) the wavelength, (c) the period, (d) the initial phase and (e) the wave speed.
- To start to address this problem compare the given wave function with the algebraic version.

$$y(x,t) = A\,sin(kx \square \square t + \square)$$

1.5 Sine Waves and Periodic Waves

$$y(x,t) = 0.02\sin(0.4x \;\square\; 50t + 0.8)m$$

$$y(x,t) = A\sin(kx \;\square\square\; t + \square)$$

(a) Amplitude A = 0.02 m

(b) Wave number k = 0.4 rad m^{-1}. As k = 2π/□ then □ = 5π m

(c) Angular frequency □ = 50 rad s^{-1}. As □ = 2πf and f = 1/T then T = 2π/w. Hence T = π/25 s.

(d) Initial phase □ = 0.8 rad.

(e) Wave speed v = □/k = 50 rad s^{-1} /0.4 rad m^{-1} = 125 m s^{-1}

1.6 The Phase of a Periodic Wave

- The term $kx - \omega t + \phi$ gives the phase of the wave.
- The phase can be understood using circular motion with the amplitude of the wave defining the radius of the circle.
- The radius will rotate counterclockwise as it traces out the circle.

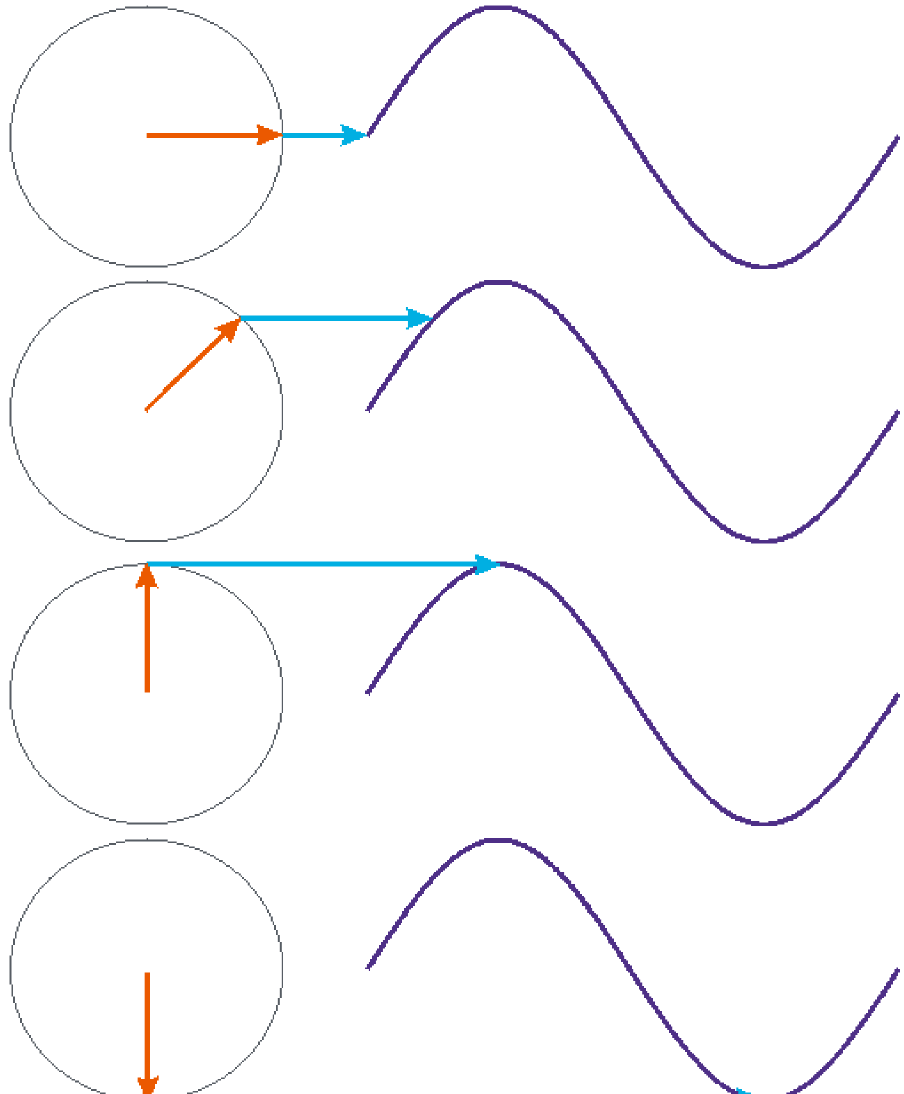
If we let *kx*- □*t* +□ be a single variable □ the wave function can be written as y(x,t) = Asin(□). As □ advances then the displacement from the x or t axis changes.

□ = 0 rad

□ = π/4 rad

□ = π/2 rad

□ = 3π/2rad



1.7 Phase difference between two points on a wave

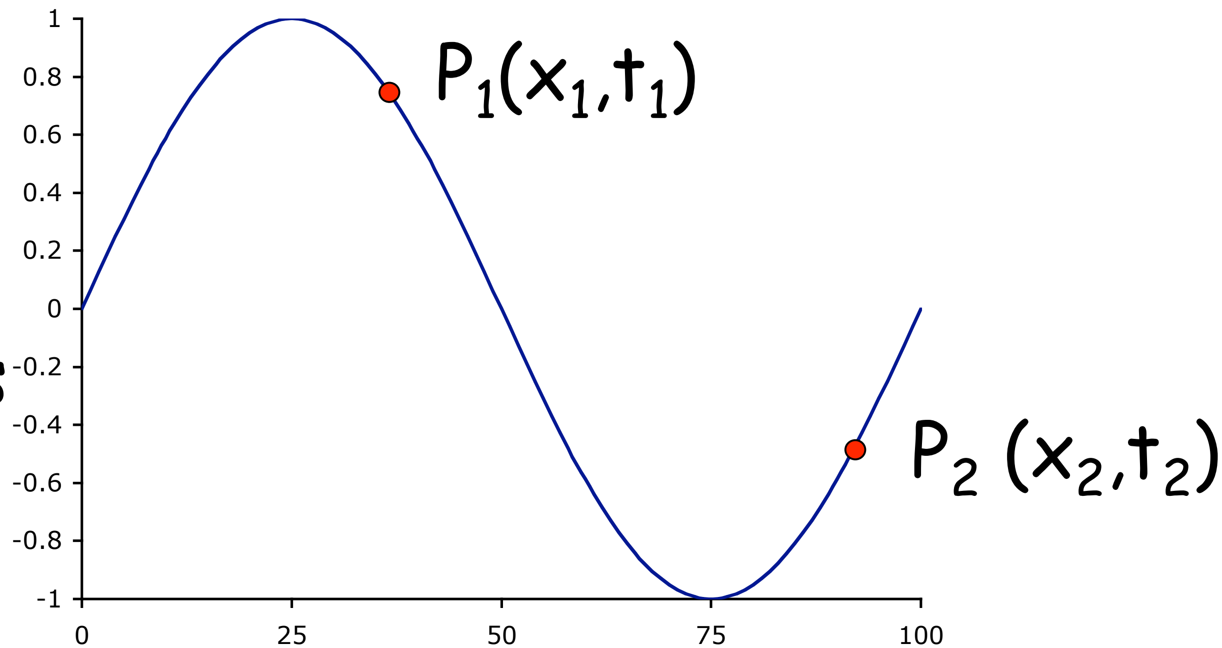
The wavelength and period define the distance and time for the wave to repeat by 2π rad. The phase difference $\Delta\phi$ between any two points on a wave is found as follows.

At P_1 the phase is

$$\phi_1 = kx_1 - \omega t_1 + \phi$$

At P_2 the phase is

$$\phi_2 = kx_2 - \omega t_2 + \phi$$



So the phase difference $\Delta\phi$ is $\Delta\phi = \phi_2 - \phi_1$

$$\Delta\phi = \phi_2 - \phi_1 = k(x_2 - x_1) - \omega(t_2 - t_1)$$

1.7 Example on Phase Difference

A harmonic wave is described by the wave function

$$y(x,t) = 0.02\sin(0.4x - 50t + 0.8)\ \text{m}.$$

Find the phase diference between two points

(a) Separated in space by 0.6 m at the same time

(b) Separated in time by 0.03 s at the same point in space

So the phase difference $\Box$ is $\Box = \Box_2 - \Box_1$

$$\Box = \Box_2 - \Box_1 = k(x_2 - x_1) - \Box(t_2 - t_1)$$

(a) Here $x_2 - x_1 = 0.6$ m and $t_1 = t_2$.

So $\Box = k(x_2 - x_1) = 0.4 * 0.6$ rad m^{-1} m = 0.24 rad.

(b) Here $x_2 = x_1$ and $t_2 - t_1 = 0.03$ s.

So $\Box = -\Box(t_2 - t_1) = 0.03 * 50$ rad s^{-1} s = 1.5 rad.

<u>1.8 The initial phase of a wave</u>

A harmonic wave is generally described by the wave function

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

To what does ϕ correspond?

Let us set x = 0m and t = 0 s.

$$y(0,0) = A\sin(\phi)$$

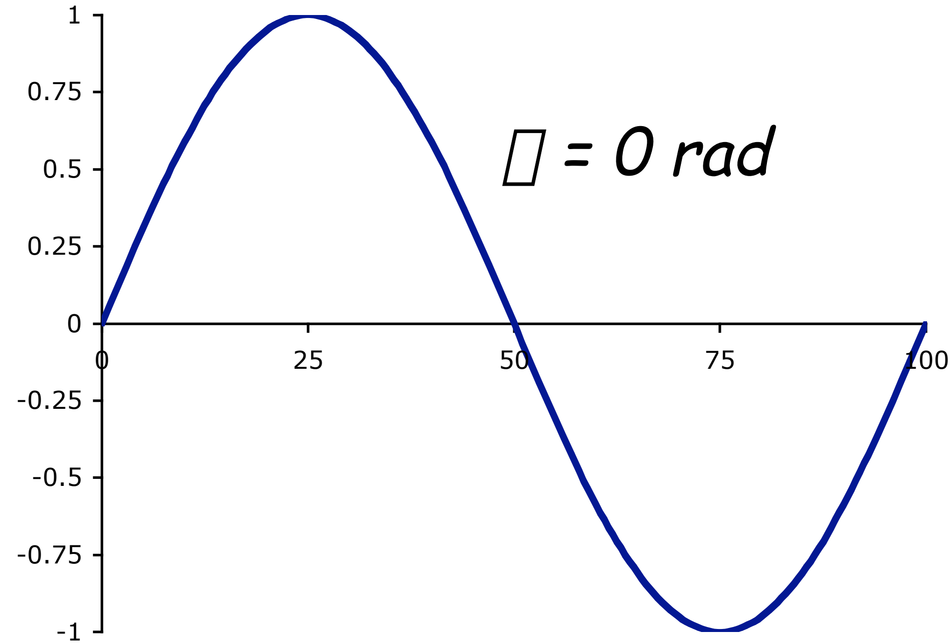
So ϕ gives the displacement of the wave at x = 0m and t = 0 s. Hence the name *- initial phase.*

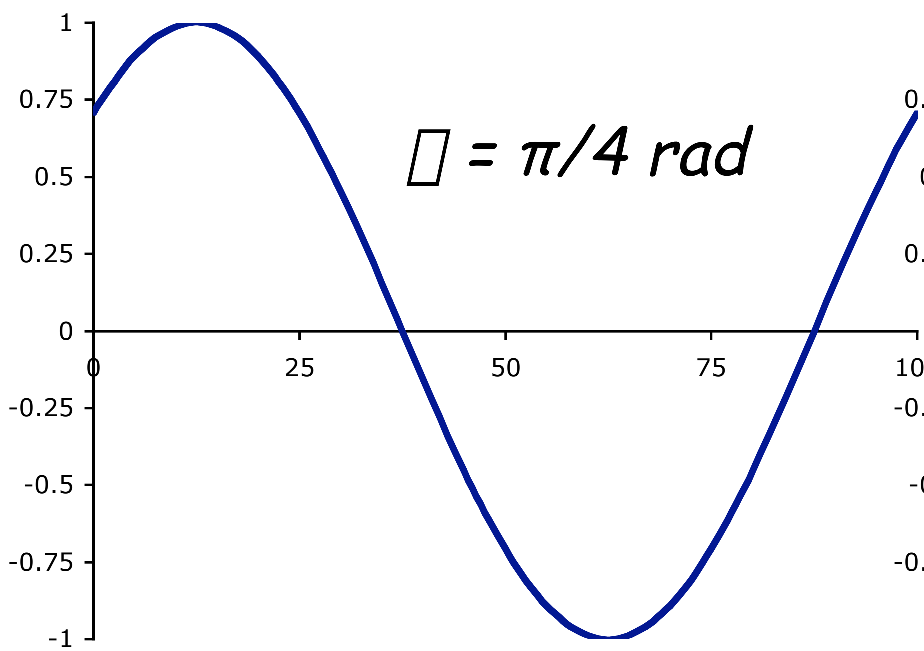
ϕ does not change the sequence of the events in a wave it only makes them happen sooner or later in the sequence.

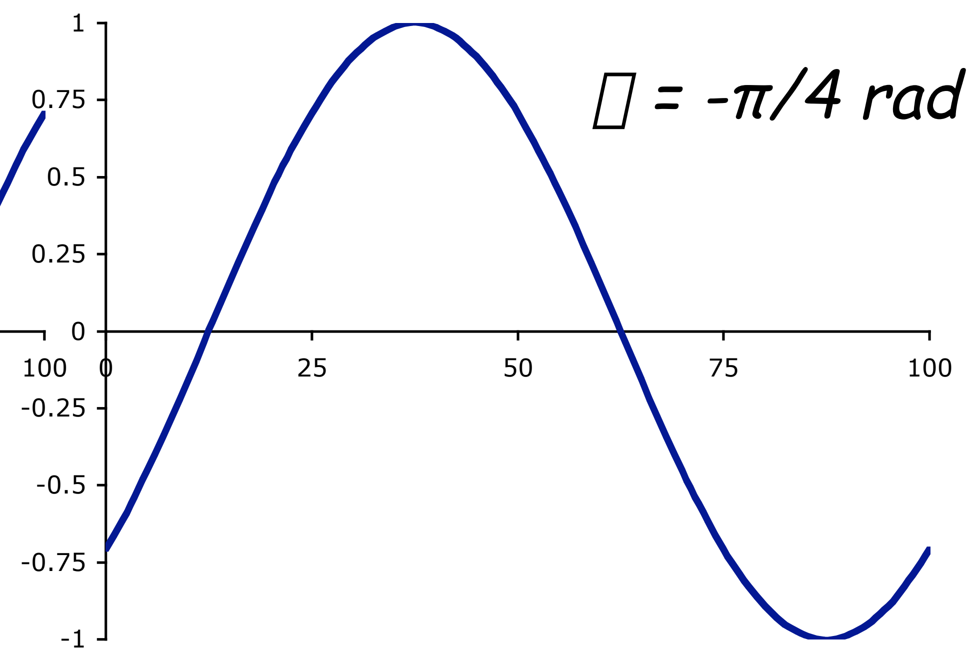
Let us consider a harmonic wave of the form

$y(x,t) = A\sin(kx - \omega t + \phi)$

For various values of ϕ







1.9 Particle motion and Harmonic Waves

- Have defined a wave as a disturbance from the equilibrium condition that propagates without the transport of matter.
- For a harmonic wave the particles oscillate in the same way as a harmonic oscillator and execute simple harmonic motion.
- Particles therefore have a
  - Particle speed v_p
  - Particle acceleration a_p

1.9 Particle motion and Harmonic Waves

- Let displacement be described by

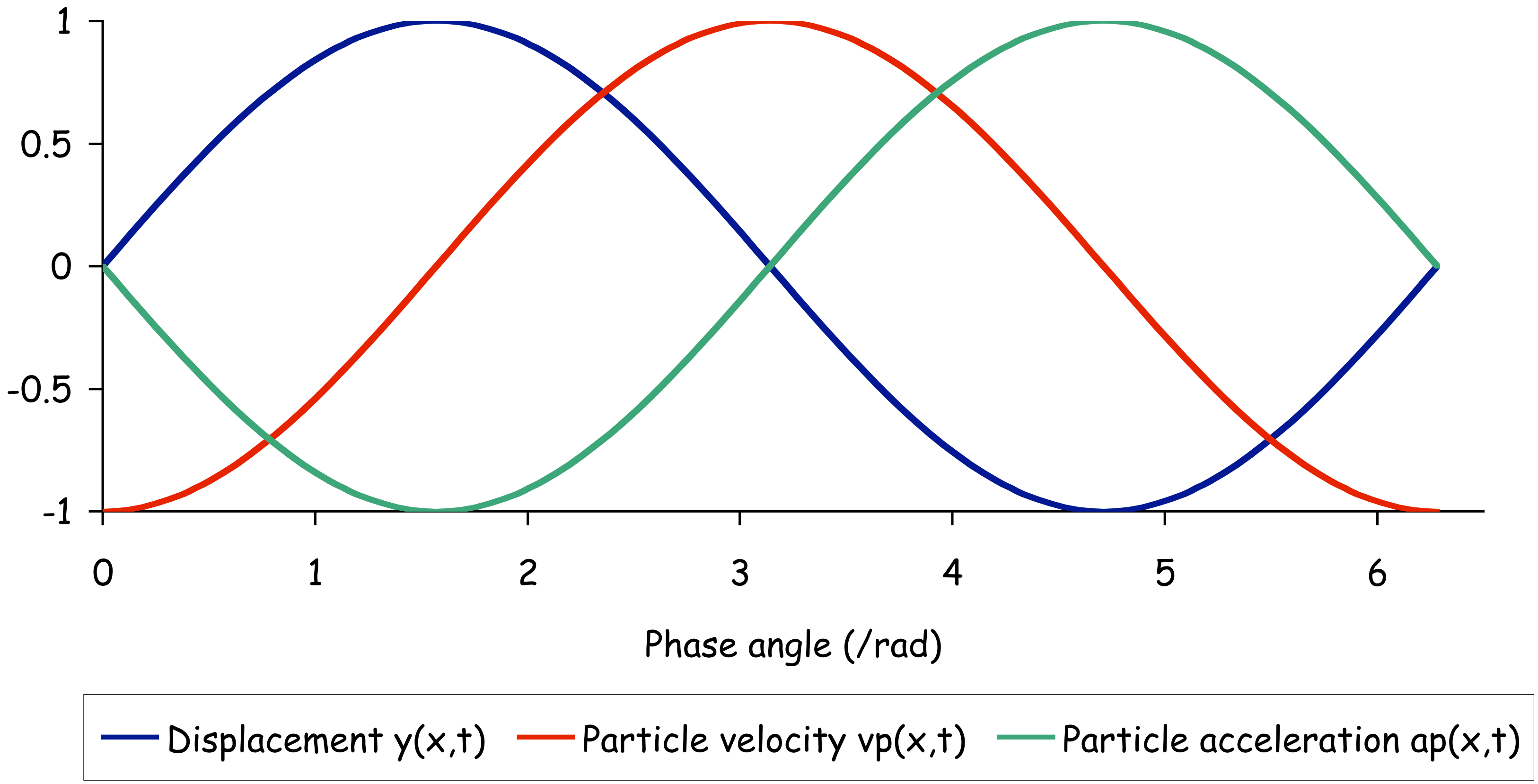
$$y(x,t) = A\sin(kx - \omega t + \phi).$$

- Particle speed $v_p(x,t)$

$$v_p(x,t) = \frac{dy(x,t)}{dt} = -\omega A\cos(kx - \omega t + \phi)$$

- Here we treat $kx + \phi$ as constants that are independent of time.
- Particle acceleration $a_p(x,t)$

$$a_p(x,t) = \frac{d^2y(x,t)}{dt^2} = -\omega^2 A\sin(kx - \omega t + \phi)$$



$v_p(x,t)$ is $-\pi/2$ out of phase with $y(x,t)$ - QUADRATURE

$a_p(x,t)$ is $-\pi$ out of phase with $y(x,t)$ - ANTIPHASE

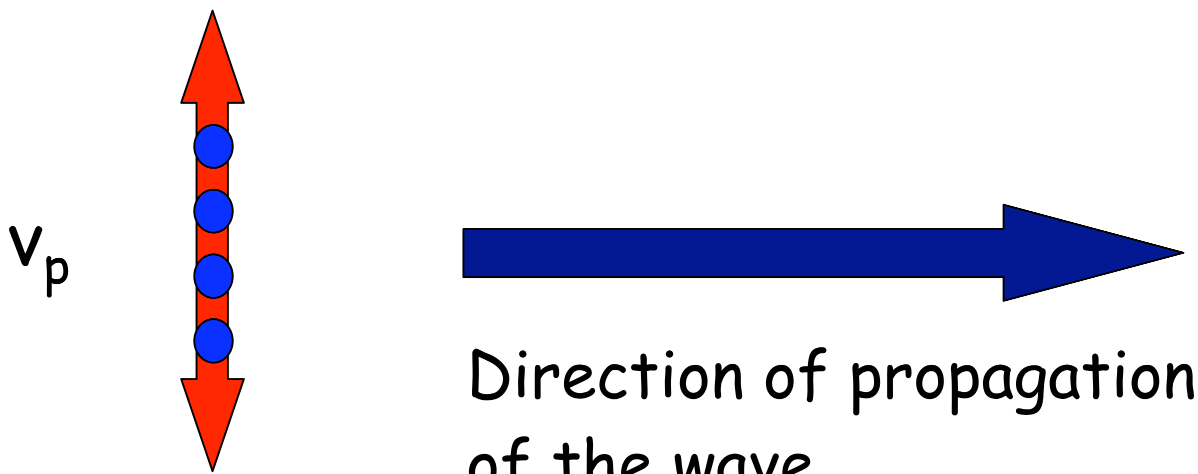
1.9 Particle motion and Harmonic Waves

- We must take care not to confuse the wave speed and the particle speed.
- The wave speed is the speed at the wave propagates through the medium.
  - $v = f\lambda = \omega/k$
- The particle speed/velocity is the speed/velocity at which the particle oscillates about its equilibrium position.

$$v_p(x,t) = \frac{dy(x,t)}{dt} = -\omega A\cos(kx - \omega t + \phi)$$

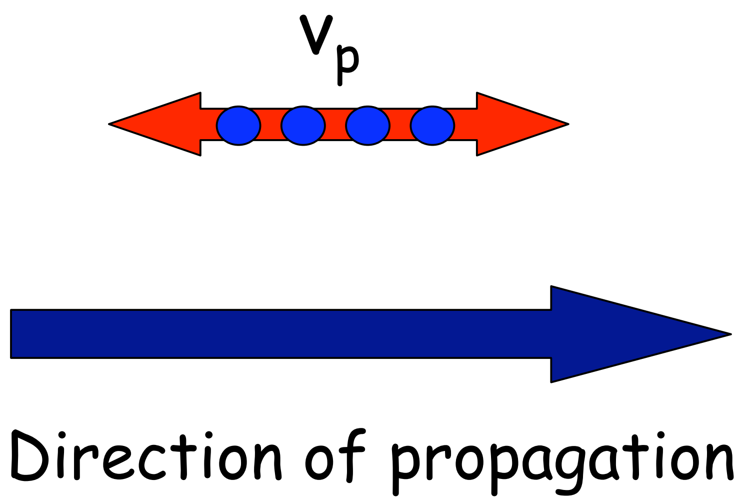
Particle speed and wave speed

- **Transverse waves**:- The displacement is at right angles to the direction of propagation. So the particle velocity v_p is at right angles to wave speed v



Direction of propagation of the wave.

Longitudinal waves:-The displacement is in the same direction as the wave propagates. So particle velocity v_p is parallel with the direction of wave speed.



1.9 Particle speed and acceleration

A harmonic wave is described by the wave function.

$$y(x,t) = 2.4 \times 10^{-3}\sin(36x - 270t) \text{ m.}$$

What is

(a) the maximum particle speed and

(b) the particle acceleration at $x = 4$ m and $t = 1$ s?

To start let us write the wave function in the standard form

$$y(x,t) = A\sin(kx - \square t)$$

For part (a): the particle velocity $v_p(x,t)$ is given by

$$v_p(x,t) = \frac{dy(x,t)}{dt} = \frac{d(A\sin(kx \square\square t))}{dt}$$

So the particle velocity $v_p(x,t)$ is given by

$$v_p(x,t) = -\square A\cos(kx - \square t)$$

Here the maximum particle speed $v_{pmax}(x,t)$ occurs when

cos(kx -wt) = -1.

$v_{pmax}(x,t) = \square A$

$v_{pmax}(x,t) = 270 \text{ rad s}^{-1} \square 2.4 \text{ X } 10^{-3}\text{m} = 0.65 \text{ m/s}.$

For part (b): the particle acceleration $a_p(x,t)$ is given by

$$a_p(x,t) = \frac{d^2 y(x,t)}{dt^2} = \frac{d^2\left(A\sin(kx - \omega t\right)}{dt^2}$$

So the particle acceleration $a_p(x,t)$ is given by

$$a_p(x,t) = -\omega^2 A\sin(kx-\omega t)$$

Here ω = 270 rad s^{-1}, k = 36 rad m^{-1} $A = 2.4 \times 10^{-3}$m, x = 4 m and t = 1 s.

$a_p(x,t) =-$ (270 rad $s^{-1})^2 \times 2.4 \times 10^{-3}$m cos(36 rad $m^{-1} \times$ 4 m - 270 rad $s^{-1} \times$ 1 s)

$a_p(x,t)$ = -165 m/s^2

1.9 Particle motion and Harmonic Waves

- A harmonic wave is described by the wavefunction

  $$y(x,t) = 0.02\sin(0.4x - 50t + 0.8)\ \text{m}.$$

- Determine (a) the wave speed, (b) the particle speed at x = 1 m and t = 0.5 s
- To solve write the wave function as $y(x,t) = A\sin(kx - \omega t + \phi)$
- k = 0.4 rad m^{-1} ω = 50 rad s^{-1}

  (a) Wave speed v = ω/k = 50/0.4 rad s^{-1} /rad m^{-1} = 125 m s^{-1}

  (b) Particle speed v_p

  $$v_p(x,t) = \frac{dy(x,t)}{dt} = -\omega A\cos(kx - \omega t + \phi)$$

- k = 0.4 rad m^{-1} ω = 50 rad s^{-1} x = 1 m and t = 0.5 s
- $v_p(x,t)$ = -50*0.02cos(0.4radm^{-1}*1m - 50rads^{-1}*0.5s +0.8rad) m s^{-1}
- $v_p(x,t)$ = 0.23 m s^{-1}