|  |  |  |  |
| --- | --- | --- | --- |
| **Lesson Title**: Factors, Multiples, Least Common Multiple, & Greatest Common Factor  **Created by:** Kathleen DeMars | | | **NRS Level of Lesson:**  NRS 3 |
| **Intended Modality:** (check all that apply)  x In-person □ Virtual □ Hybrid | | | |
| **Content Area(s)** | **Targeted** [**IL ABE/ASE Content Standards**](http://www.excellenceinadulted.com/resources/abease-curriculum-project/) | | |
| **3.OA.4 (supporting)** | Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. | | |
| **3.OA.5 (supporting)** | Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9+2). | | |
| **Integrated** [**Essential Employability Skills**](https://www.illinoisworknet.com/DownloadPrint/ILEssentialEmployabilitySkills-Handout.pdf) | | | |
| □ Personal Ethic *(Integrity, Respect, Perseverance, Positive Attitude)* | | **x** Teamwork *(Critical Thinking, Effective & Cooperative Work)* | |
| □ Work Ethic *(Dependability, Professionalism)* | | **x** Communication *(Active Listening, Clear Communication)* | |
| **Lesson Objectives *(Students will be able to)****:*   * **Students will be able to answer this essential question:**   + How do we use our understanding of factors and multiples to find the greatest common factor and the least common multiple?     - Furthermore, how can we apply finding the greatest common factor and the least common multiple to real-world scenarios?      * Define, identify, and find factors, multiples, greatest common factors, and least common multiples in computation problems.      * Solve real world problems requiring we find the least common multiple or the greatest common factor that may include the distributive property. | | | |
| **Engagement is not “one size fits all.” How are you providing multiple ways to engage all learners? Click on** [**Multiple Means of Engagement**](https://udlguidelines.cast.org/engagement) **to learn more about providing options for learners and explain how you are including this below:**   * We will **minimize threats and distractions** by creating a supportive and accepting classroom environment, including all students in class discussion, and varying the risk students face when completing tasks by giving them opportunity to scaffold prior knowledge, work together, and benefit from direct instruction. | | | |
| **Key Vocabulary**:  **Factor:** Numbers that we multiply together are called factors.  **Multiple:** The product of a multiplication problem is also a multiple of each of the factors.  **Greatest Common Factor:** The largest factor that two or more integers share is called the greatest common factor.  **Least Common Multiple:** The smallest multiple that two or more integers share is called the least common multiple.  **Distributive Property:** When we can pull a factor from two or more integers and then create an expression that can then be multiplied by the pulled factor, we are implementing the distributive property. Multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together. | | | |
| **Instructional Materials:**  Textbooks or online curriculum: Slide show with notes and application opportunities embedded. Printable final 4 slides for station work.  Websites: None | | | |
| **Lesson Activities:**  ***Slide 2:***  *Factors are numbers that we multiply together to get another number. Most numbers have more than two factors and often the larger the number, the more factors it will have.*  *Looking at the example on this slide, we can multiply 3 x 7 to make 21. We can also multiply 1 x 21 to make 21. Each of those pieces are called factors. So, 1, 3, 7, and 21 are all factors of 21. We can multiply any of those factors by another number in the factor list to create 21.*  ***Slide 3:***  *Here we have an opportunity to find the factors of three different numbers. Let’s take a minute to work with a partner if you’d like to come up with the factor pairs for A, B, and C. By factor pair, we mean which two numbers can we multiply together to create the multiple of 20, 36, and 45.*  *Tip: Start with 1 and work your way up. We can always multiply 1 times the number to create a factor pair. Each number in the pair is a factor.*  ***Slide 4:***  *Let’s look at letter A, which asked us to find the factors of 20. If we start with 1, we can see that 1 x 20 = 20. I recommend starting with 1 and working your way up so that you can be sure that you catch all of the factors. Finding factors in an orderly way is noticing the structure to our work, which is one of our Standards for Mathematical Practice.*  *As we move up from 1 to 2, we ask ourselves, “can we multiply anything times 2 to get a product of 20?” YES! We can multiply 2 x 10 to create a product of 20. Moving along, we may start to recognize which numbers are going to work as factors and which we can skip because we are becoming fluent with our multiplication facts. However, if we are not quite fluent, we can move up the number line one by one and ask ourselves the same question until we have climbed all the way to 20. For example, can we multiply anything times 3 to create 20? You may think to yourself, 3 times 7 is 21 and that’s too big. 3 x 6 is 18 and that’s too small, so there is no whole number that we can multiply times 3 to create a product of 20. Therefore, 3 is not a factor of 20.*  *Once we have found all of our factor pairs: 1 x 20, 2 x 10, and 4 x 5, we can see that 1, 2, 4, 5, 10, and 20 are all factors of 20. Each of these numbers on its own can be multiplied times another number to create a product of 20.*  ***Slide 5:***  *In letter B, we are asked to find the factors of 36. We can use the same strategy and process that we used in letter A to accomplish this goal.*  *Where should we begin? – wait for students to suggest starting with 1. This indicates that they are noticing the structure of the problem.*  *After students indicate 1 x 36 = 36, ask for what we should go to next? If no one responds, ask what comes after 1. When students reply, “2” ask them if we can multiply anything times 2 to make 36? Many students may not respond because they don’t know their 18 times tables, and that’s ok! Remind students that multiplication and division are inverse operations. To help them navigate this problem, you might say, “can 36 be divided by 2?” Some students should answer yes. Ask them “how do you know?” Students will likely reply that 36 is an even number and even numbers can be divided by 2. From there, we can ask, “what is 36 divided by 2?” The answer is 18, so 2 x 18 = 36 because 36 / 2 = 18. Point out inverse operations as a viable strategy for solving for factors.*  *Move along up the number line eliminating 5 as an option that will not work. Point out that numbers that are divisible by 5 end in 0 and 5 to assist students with recognizing patterns.*  *Finally, make a point of telling students that “1, 2, 3, 4, 6, 9, 12, 18, and 36 are all factors of 36 because each of these numbers can be multiplied by another number to create 36.” This repetition from slide 4 is important. We will see it again on slide 6. We are reinforcing that we recognize the pattern-like approach to solving for factors.*  ***Slide 6:***  *Finally, we have letter C, which asks us to find the factors of 45. By now you have figured out that finding the factors means asking ourselves which numbers can be multiplied together to make 45. But you’ve also figured out that this means, what numbers can I divide 45 by? Using inverse operations to solve problems is a sign of thinking like a mathematician!*  *The factors of 45 are 1, 3, 5, 9, 15, and 45. Each of these numbers can be multiplied times another number to make 45. Similarly, 45 can be divided by each of these numbers.*  ***Slide 7:***  *We have talked about factors, and you’ve heard me mention the word multiple and product interchangeably. What is a multiple?*  *●A multiple is the number or result/answer that we get from multiplying two or more factors together.*  *For example, in red we see 4 x 5 = 20. 20 is a multiple of 4 and 20 is a multiple of 5. If we skip count by 4’s we will reach 20 after 5 skips. If we skip count by 5s we will reach 20 after 4 skips.*  *When we multiply integers together, we get a multiple.*  *●An integer is any positive or negative whole number.*  *Any integer has infinite multiples. We can skip count forever and never run out of multiples.*  ***Slide 8:***  *Let’s practice finding multiples. Remember, unlike factors, there are not a set number of multiples. Multiples can go on forever to infinity. So, we won’t be finding all of them; it would be impossible! Instead, let’s list 3 multiples of each of the numbers listed in A, B, and C.*  *You can start at the number and skip count by that number, or you can multiply the number times any three other numbers you choose. Both ways you’ll come up with a multiple.*  *For example, I may choose to skip count by 4s for letter A. For letter B, I may choose to multiply 7 times 100, 7 times 7, and 7 times 10. This would also create three multiples of seven. The choice is yours because there are infinite multiples.*  *\*\* Pause here and allow students time to work on their own or with a partner to find at least 3 multiples for each number listed. Allow 3 minutes.*  ***Slide 9:***  *Here are some possible answers that your team may have come up with! Notice that I chose to skip count for each of the problems. Would it be correct if I had chosen to multiply by much larger numbers?*  *-Yes, it would have been correct because there are infinite multiples. These are only EXAMPLES of correct responses.*  *When you look at these three rows of multiples, do you notice any multiples that are the same from row to row?*  *\*\* Allow time to look at the lists\*\* - 15-20 seconds. Ask for student responses.*  *Possible responses: 28, 63, and 36 each appear in 2 rows of multiples.*  ***Slide 10:***  *You noticed that 28 appears in rows A and B.*  *You noticed that 36 appears in rows A and C*  *You noticed that 63 appears in rows B and C*  *This means that each of these numbers that appear in multiple rows are COMMON MULTIPLES. This means that 28 is a multiple of 4 AND 28 is a multiple of 7. 36 is a multiple of 4 AND 36 is a multiple of 9. 63 is a multiple of 7 AND 63 is a multiple of 9. These are COMMON MULTIPLES.*  *Common multiples are multiples that two or more integers share. Share means that they are the SAME.*  ***Slide 11:***  *Now that we know that numbers can have multiples in common, let’s talk about the LEAST COMMON MULTIPLE.*  ●*Least Common Multiple is often denoted as LCM. This is the LOWEST multiple that two or more numbers have in common. In our example here, we noticed that 4, 7, and 9 had a handful of common multiples.*  ○*4 and 7 had 28 in common*  ○*4 and 9 had 36 in common*  ○*7 and 9 had 63 in common*  *The LEAST COMMON MULTIPLE is the first time that two numbers have a multiple in common. We already know that each number has infinity multiples. We can never count them all. But, we can notice the first time that two numbers have a multiple that is the same and this is called the LCM.*  *A trick to finding the LCM is to skip count by the higher of the two factors. When you come to one that is a multiple of both that you’re considering, you’ve found the Least Common Multiple.*  *Let’s give it a try.*  ***Slide 12:***  *Let’s try letter A together \*\* Scaffold the material for students by using repeated direct instruction. Then, allow students to work in pairs or trios so that they can use their own words to explain their thinking to one another.\*\**  *In Letter A, we have the factors 3 and 4 and we want to know what the least common multiple is. Remember, this means the first time that these two numbers have a multiple in common.*  *My trick was to skip count by the higher of the two factors so let’s try that for letter A and see what happens.*  *Skip count by 4: 4, 8, 12, 16, 20 (It may help to write these numbers on the board). Pause and ask: “we have skip counted 5 times, are any of these numbers multiples of 4 as well? If there is silence, rephrase by asking, “are any of these multiples of four also divisible by 3?”. Remind students that we can use our understanding of inverse operations to help us think critically about the problem we are solving.*  *Answer: 12 is a multiple of 3 and a multiple of 4. 12 is the first multiple that is the same for both 3 and 4.*  *NOTE: Students may ask, “Can we just multiply the factors together to find the LCM? 3 x 4 = 12.  Applaud students for this thinking. Then, explore if they’re correct. While it is true that we can*  *ALWAYS find a common multiple between integers by multiplying them together, and this is a skill we may sometimes rely upon when working with fractions to find common denominators in the future, it is not a sure fire way to find the least common multiple. Give the example of 6 and 12. If we multiply 6 x 12 we will get 72. 72 is absolutely a multiple of both 6 and 12. However, the LCM of 6 and 12 is actually just 12. 6 x 2 = 12 and 12 x 1 = 12, so while the student is correct in using this strategy to find a common multiple, it will not be consistently successful in finding the least common multiple.*  *Once you’ve worked through letter A as a large group, encourage students to partner up in pairs or trios to work on letters B and C*  ***Slide 13:***  *To review, here are our multiples of 3 and our multiples of 4. From our list of multiples we can see that the first time that 3 and 4 share a multiple that is the same is when we reach 12. You might notice that they also share 24 and that is a COMMON multiple. However, it is not the LEAST or LOWEST common multiple of 3 and 4.*  *Reiterate the point that there are infinite common multiples.*  ***Slide 14:***  *In letter B, we were asked to find the LCM of 5 and 6. In this slide we can see the multiples listed in ascending order starting from the smallest. We skip count by 5s and we skip count by 6s until we see a multiple that is the same.*  *Again, it is important to note that we cannot reliably find the LCM by multiplying two numbers together. While that will generate a common multiple, it is not guaranteed to be the lowest or least common multiple.*  *The least common multiple of 5 and 6 is 30.*  ***Slide 15:***  *Finally, let’s take a look at our 3rd problem, letter C. Here we were asked to find the LCM of 7 and 8. Again, you can see the list of multiples starting with the lowest for both 7 and 8. Again, we see here that the LCM was the product of 7x8, just like in A and B. I want to emphasize AGAIN that this will not always be the case. Multiplying two integers together will always yield a multiple, but it will not always be the least common multiple. Please take a moment to write that down*  *--- Write on board: multiplying two numbers together will always give us a multiple, but it is NOT always the least common multiple (LCM).*  *Example: the LCM of 6 and 12 is 12, NOT 72.*  *In letter C, the LCM of 7 and 8 is 56. 56 is the first multiple that is the same for both 7 and 8.*  ***Slide 16:***  *REMEMBER: Factors are two integers that are multiplied together to create a multiple or product. Two numbers can have factors in common just like they have multiples in common. To find the factors that a pair of integers have in common, we list each number’s factor pairs and compare them!*  *For example: if we have the numbers 25 and 15 and we want to find the common factors, we would start by listing all of the factor pairs for 25 and all of the factor pairs for 15. As we can see in the slide, the factor pairs of 25 are 1 x 25 and 5 x 5. This means that 1, 5, and 25 are the factors of 25. We do the same thing for 15. The factor pairs of 15 are 1 x 15 and 3 x 5. This means that 1, 3, 5, and 15 are the factors of 15.*  *The largest or greatest factor that is the same for 25 and 15 is 5. This is the highest common factor for both 25 and 15.*  *The greatest common factor, or GCF is the highest factor that two or more numbers have in common.*  *Another way to think about the GCF is to consider what is the LARGEST number that divides evenly into both of these numbers? That is the GCF.*  ***Slide 17:***  *This is important for students to write down. Please repeat and emphasize this definition.*  *The GCF (Greatest Common Factor) is the highest factor that two or more numbers share.*  *Now, let’s practice finding the GCF as a whole class and then in our small groups*  ***Slide 18:***  *Let’s work on letter A as a class together.*  *It asks us to find the GCF, which stands for Greatest Common Factor, of 20 and 30. Remember that the greatest common factor is the biggest factor that two or more numbers have that is the same. Another way to think of the GCF is to consider what is the biggest number that goes into both 20 and 30 evenly?*  *To find the GCF of 20 and 30, we can list the factor pairs of 20 and the factor pairs of 30. Then we can create a list from smallest to largest of each of the factors for 20 and each of the factors for 30. Then we can compare the lists to see what the highest factor is that is the same on both lists.*  *Remember: numbers may share more than one factor in common. The GCF asks us which common factor is the biggest!*  *Factor pairs of 20:*  *1 x 20, 2 x 10, 4 x 5*  *Factors of 20: 1, 2, 4, 5, 10, 20*  *Factor pairs of 30:*  *1 x 30, 2 x 15, 3 x 10, 5 x 6*  *Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30*  *The largest factor that 20 and 30 have that is the same is 10.*  *NOTICE: There are other common factors: 1, 2, and 5 are all common factors, but 10 is the greatest common factor.*  *Now try letters B and C with your small group.*  ***Slide 19:***  *While you’re working with your partner on letters B and C, you can see how we solved letter A together if you need a point of reference.*  ***Slide 20:***  *Now, let’s take a look at letter B together.*  *What did your brain tell you to do first?*  *-possible response: find the factor pairs of 39 and find the factor pairs of 13.*  *What did your brain tell you to do next?*  *-list the factors in order from smallest to largest for both numbers.*  *What did your brain tell you to do last?*  *-Look at the two lists of factor pairs and notice the largest match. This is the GCF*  *Is there any other approach or strategy we could have used to find the GCF?*  *-We could have asked ourselves, “what’s the largest number that I can divide both 13 and 39 by without any remainder? The answer would have been 13.*  ***Slide 21:***  *Let’s look at our final computation practice problem before we move onto applying the concepts of LCM and GCF to real world scenarios. That’s the fun stuff!*  *In our final computation problem we were asked to find the GCF of 48 and 16. There are two ways we could have approached this. The first is shown on the slide. We can list the factor pairs for each number. Then, we can list the factors from smallest to largest for each number. Finally, we can compare those two lists and pick out the biggest factor that both numbers share. The alternate way to find the GCF of 48 and 16 would be to ask ourselves, what is the largest number that we can divide both 48 and 16 by without any remainder. The largest divisor of 48 and 16 is 16. 48 / 16 = 3 and 16 / 16 = 1*  *Please notice that there are lots of factors that are the same for 48 and 16. There are 1, 2, 4, and 8 that are all common factors. The GCF wants to know which factor is the BIGGEST and the biggest is 16.*  ***Slide 22:***  *The next 4 slides have real world problems. These can be printed out and set up as stations in the classroom so that students have the opportunity to move around while they learn. These problems can be solved in pairs/trios to allow students to share their thinking out loud. We learn when we teach. This will also give an opportunity for formative assessment. The instructor can circulate around the room between stations to listen and observe how students are solving each problem. In the notes of each of the slides is the solution that can be shared as a large group once the application activity is complete. The students should be working together on their own with very limited teacher help for this activity. We would like to se them apply what they’ve learned in real world situations.*  ***Slide 23:***  *ANSWER:*  *We are looking for the Least Common Multiple (LCM)*  *President: 4, 8, 12, 16, 20*  *Senator: 6, 12, 18, 24*  *The LCM is 12.*  *This means that if I voted for both a senator and the president this year, the next time I will have the opportunity to vote for the president and senator again will be in 12 years time.*  ***Slide 24:***  *ANSWER*  *We are looking for the Least Common Multiple because we want to know the next closest time that the comets will pass on the same year and then we want to know which year it will be.*  *Comet A: 5, 10, 15, 20, 25, 30, 35, 40*  *Comet B: 7, 14, 21, 28, 35, 42, 49, 56*  *The first time that both Comet A and Comet B will pass during the same year is 35 years after the last time they were together. The last time they were together was in 2020. So, we must add 35 years to 2020. The next time that the comets will pass during the same year is in 2055. 2020 + 35 years = 2055.*  ***Slide 25:***  *ANSWER:  13 students. Each student will receive 3 pencils and 2 pens.*  *13 (3 + 2)*  *We are looking for the GCF of 26 and 39. Then, we are using the GCF to figure out how many of each item the students will receive.*  *The largest number that 39 and 26 can be divided by is 13. This means that the largest number of students in the class is 13. 13 x 2 = 26. Since there are 26 pens, each student will get 2. 13 x 3 = 39. Since there are 39 pencils, each student will get 3 pencils.*  *Draw attention to the use of the distributive property when we identified how many pens and pencils each of the 13 children would receive in their Valentine gift.*  ***Slide 26:***  *ANSWER:*  *Rafi + 20 guests = 21 people at the BBQ.*  *21 x 2 = 42. This is how Rafi figures that he needs at least 42 hotdogs and buns.*  *We are solving for LCM and then applying real life common sense to find the answer.*  *Hot dogs = 6 per pack*  *Buns = 10 per pack*  *He needs to have at least 42 hotdogs and 42 buns and doesn’t want any of either left over without a match meaning no lonely hotdogs without a bun or buns without a hotdog.*  *Hotdogs: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72*  *Buns: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100*  *The LCM is 60. HOWEVER: This does not mean that Rafi should buy 6 packs of hotdogs and 60 packs of buns. That would be way too many and it would leave a lot of buns without hotdogs to fill them. Instead, we need to look at what the factor pair was to make 60 for both hotdogs and buns.*  *Hotdogs: 6 in a package x 10 packages = 60 hotdogs …..* ***we need 10 packages of hotdogs***  *Buns: 10 in a package x 6 packages = 60 buns……..* ***we need 6 packages of buns***  *10 packages of hotdogs and 6 packages of buns will give us at least 42 hotdogs and hotdog buns and it will give every hotdog available a bun.* | | | |
| **Learners vary in the way that they react to and grasp information that is presented to them. Click on** [**Multiple Means of Representation**](https://udlguidelines.cast.org/representation) **to explore ways that you can provide options for representing content and explain how you are including this below:**   * We **will clarify vocabulary and symbols** by explicitly teaching relevant vocabulary to the lesson through repeated direct instruction. We will also lean into learners’ experience and prior knowledge * We will **offer ways of customizing ways of displaying information** by allowing students to view content on the large screen as well as in front of them on handouts that they may write on. Additionally, students will be able to engage with the application activities moving around the room or individually in their own space. * We will **support decoding of text, mathematical notation, and symbols** by offering clarification of key terms ahead of application. * We will **maximize transfer and generalization** by providing explicit, supported opportunities to generalize learning to new situations through our application activity at the end of the lesson. | | | |
| **Performance Tasks:**   * Opportunities to practice finding factors, multiples, least common multiples, and greatest common factors are embedded within the direct instruction of the slides. These may be done as a whole class and/or as partner work with direct instruction review of each problem. * At the end of the slides there are 4 application problems that may be printed out and stationed around the classroom. Students may move from station to station solving problems either individually or in pairs/small groups. Encourage discussion of how we approach these problems. | | | |
| **Learners best express what they know in different ways. Click on** [**Multiple Means of Action & Expression**](https://udlguidelines.cast.org/action-expression) **to explore ways to offer options for learners and explain how you are doing this below:**   * We will **vary the rates for response and navigation** by allowing students to work in partners, individually, or small groups. They may express their thinking through verbal or written language. * We will **build fluencies with graduated levels of support for practice and performance** by providing scaffolds that can be gradually released with increasing independence and skills as students progress through each micro-practice set and the culminating application activity. * We will **support planning and strategy development** by embedding prompts such as, “what does your brain tell you to do first? Next? Etc.” as students move through their application activities. | | | |
| **Notes:**  **Math Practice(s) taught and practiced by students:**  **MP1:** Make sense of problems and persevere in solving them  **MP4:** Model with mathematics  **MP6:** Attend to precision  This lesson has options for flexibility in how students work and learn. Much of this lesson can be done using direct instruction and whole class learning. Alternatively, this lesson can be adapted to provide multiple opportunities for independent and partner work. Finally, this lesson may be extended by encouraging students to write their own real world  problems where one needs to find the GCF or the LCM to solve the problem. | | | |