

Answer Key

Applying Our Knowledge: Surface Area and Prisms

Directions: solve the following 3 questions.

1. How does one find the surface area of a right prism?
 $2lw + 2lh + 2wh$ (Note: students may explain this using words or the formula)
2. David bought a dictionary for his English tutor as a gift. Help David find the surface area of the dictionary so that he knows how much wrapping paper to buy. Assume all sides are flat, meaning that the cover and back have no overhang. What is the surface area of the dictionary?

Goal: What is the surface area of the dictionary?

Efficient Formula: $2lw + 2lh + 2wh$

Total Area as a sum (Surface Area): $54 + 54 + 18 + 18 + 12 + 12 = 168$ sq. in.

Surface Area using efficient formula: $2(54) + 2(18) + 2(24) = 168$ sq.in.

Guiding Questions:

- a. What will you do first? Write a sentence to explain your thinking. ***Answers could be given in a different order.**

Cover area: $6\text{in} \times 9\text{in} = 54$ sq. in.

Back cover area: $6\text{in} \times 9\text{in} = 54$ sq. in.

- b. What will you do next? Write a sentence to explain your thinking.

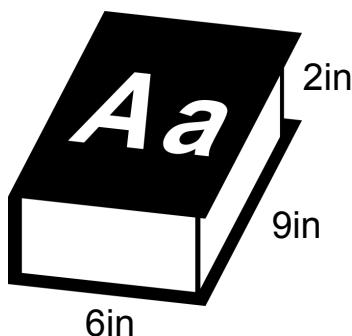
Long side area spine: $9\text{in.} \times 2\text{in} = 18$ sq. in.

Page side area opposite spine: $9\text{in.} \times 2\text{in} = 18$ sq. in.

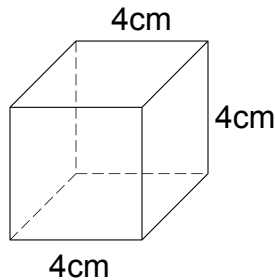
- c. What will you do last? Write a sentence to explain your thinking.

Foot/Base of book: $6\text{in.} \times 2\text{in.} = 12$ sq. in.

Head/Top of book: $6\text{in} \times 2\text{in.} = 12$ sq. in.



3. A. Find the surface area of the image below.
B. Do you notice anything about this shape that is similar or different from the shape in question 1? Share your thoughts.



- A. Area of the image using a totaling strategy and using an efficient formula strategy for finding the surface area of a right prism ($2lw + 2lh + 2wh$).**

Area of bottom: $4\text{cm} \times 4\text{cm} = 16 \text{ sq. cm.}$

Area of top: $4\text{cm} \times 4\text{cm} = 16 \text{ sq. cm.}$

Area of front: $4\text{cm} \times 4\text{cm} = 16 \text{ sq. cm.}$

Area of back: $4\text{cm} \times 4\text{cm} = 16 \text{ sq. cm.}$

Area of left side: $4\text{cm.} \times 4\text{cm.} = 16 \text{ sq. cm.}$

Area of right side: $4\text{cm.} \times 4\text{cm.} = 16 \text{ sq. cm.}$

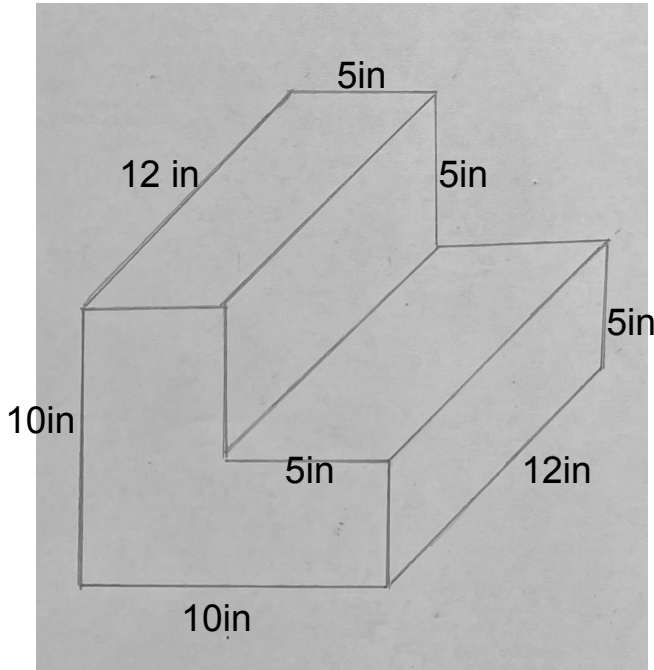
Total Area as a sum (Surface Area): $16 + 16 + 16 + 16 + 16 + 16 = 96 \text{ sq. cm.}$

Surface Area using efficient formula: $2(16) + 2(16) + 2(16) = 96 \text{ sq. cm.}$

- B. Did you notice anything? Possible answers that will show depth of understanding and mathematical flexibility with this concept:** Students may notice that all lengths of all sides were the same (4cm) in the question. They may have noticed that this is a cube, not a rectangular prism or right prism. They may have noticed that the formula for the surface area of a cube can be the same as that of a right prism and they may have noticed that the formula for the surface area of a cube could be made even more efficient than that of a right prism!

Surface area of a cube = $6(\text{length} \times \text{width})$

4. Find the surface area of the step stool. (Hint: the stool is made of two rectangular prisms stacked on top of one another). ***This problem represents an extension of the original standard.**



The students may have broken the steps into two prisms vertically or horizontally. They have enough information to successfully solve either way.

Surface area of a right prism efficient formula: $2(lw) + 2(lh) + 2(wh)$

Rectangle one (horizontal bottom step):

Area of base and corresponding top of base: $10\text{ in.} \times 12\text{ in.} = 120\text{ sq. in.}$ --- multiply this times 2 to get the area of the top of the bottom right prism.
 $2(120) = 240\text{ sq. in.}$

Area of long right side and long left side of bottom step:

$12\text{ in.} \times 5\text{ in.} = 60\text{ sq. in.}$ --- multiply this times 2 to get the area of both long sides of the bottom right prism.

$$2(60) = 120 \text{ sq. in.}$$

Area of front and back sides of bottom step rectangular prism:

10in. x 5in. = 50 sq. in. --- multiply this times 2 to get the area of both the front and back sides of the bottom step rectangular prism.

$$2(50) = 100 \text{ sq. in.}$$

Add together all 3 totals to find the surface area of the bottom **full** rectangular prism:

$$100 \text{ sq. in.} + 120 \text{ sq. in.} + 240 \text{ sq. in.} = 460 \text{ sq. in.}$$

Now, subtract half of the top's surface area because it is covered by the top step. Area of top = 120 sq.in.

$$120 / 2 = 60 \text{ sq. in.}$$

Next, subtract 60 sq. in. from the total surface area:

$$460 \text{ sq. in.} - 60 \text{ sq. in.} = 400 \text{ sq. in.} \text{ ** Hold onto this total until the end**}$$

Rectangle two (horizontal top step):

Area of long top and bottom: 12in. x 5in. = 60 sq.in.

Multiply this times 2 to find the area of the long top and bottom of the top right prism step.

$$2(60) = 120 \text{ sq.in.}$$

Area of left and right long sides: 12in x 5in = 60 sq. in.

Multiply this times 2 to find the area of the long left and right sides of the top right prism step.

$$2(60) = 120 \text{ sq.in.}$$

Area of the front and back short sides of the top right prism step:

$$5 \text{ in.} \times 5 \text{ in.} = 25 \text{ sq. in.}$$

Multiply this times 2 to find the area of the front and back short sides of the top prism step.

$$2(25) = 50 \text{ sq. in.}$$

Add together all 3 totals to find the surface area of the top full rectangular prism:

$$120 \text{ sq. in.} + 120 \text{ sq. in.} + 50 \text{ sq. in.} = 290 \text{ sq. in.}$$

Now, subtract the bottom surface area because it is covered by the other step: Area of long bottom piece = 60 sq.in.

Next, subtract 60 sq. in. from the total surface area:

$$290 \text{ sq. in.} - 60 \text{ sq. in.} = 230 \text{ sq. in.} \text{ ** hold onto this for the final answer**}$$

FINALLY: Combine the two modified surface areas that account for the overlap of steps for a total surface area.

$$\text{ANSWER: Surface area of the steps: } 400 \text{ sq. in.} + 230 \text{ sq. in.} = 630 \text{ sq. in.}$$

IMPORTANT: The most common mistake will be that the students add the surface areas of the top prism and the bottom prism together to get the total surface area of the steps. They will need to delete the area of the bottom of the top step and the area of half the top of the bottom step to get an accurate total surface area. This is **very much** an extension question that will demonstrate **thorough** conceptual understanding of surface area in the learner. It is not expected that all learners will be able to fully solve this problem correctly, particularly the last part where they'll need to make deletions. It can be said that if the students break apart the steps into two steps and solve for the surface area of each right prism correctly, that they have a solid grasp of this content standard. If they solve including the deletions, they are advanced.